Graphic Realizations of Joint-Degree Matrices

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In this paper we consider a modification of the classical *degree sequence graphic* (resp. connected graphic) *realization* problem studied by Erdős-Gallai and Havel-Hakimi, that arises naturally in networking topology generation applications. We define the *joint-degree matrix graphic* (resp. connected graphic) *realization* problem where, in addition to the degree sequence, the exact number of desired edges between vertices of different degree classes is also specified. Let $V = [n]$ be a set of vertices. Let $\mathbb{V} = \{V_1, V_2, \ldots, V_k\}$ be a partition of V denoting subsets of vertices with the same degree. Let d be function $d : \mathbb{V} \to \mathbb{N}$ denoting the degree of vertices in class V_i. Let $D = (d_{ij})$ be a $k \times k$ matrix denoting the number of edges between V_i and V_j. The problem is, given $\langle \mathbb{V}, d, D \rangle$, decide whether there is a simple (connected) graph G on V, such that, $\forall i$, each vertex in V_i has degree $d(V_i)$, and, $\forall i \neq j$, there exactly d_{ij} edges between V_i and V_j, while, $\forall i$, there are exactly d_{ii} edges entirely inside V_i. We give polynomial time decision and construction algorithms for the graphic and connected graphic realization problems.

Joint work with Bradley Green, and Milena Mihail.