Number of Crossing-Free Geometric Graphs vs. Triangulations

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We show that there is a constant $\epsilon > 0$ such that, for any set P of $n \geq 5$ points in general position in the plane, a crossing-free geometric graph on P that is chosen uniformly at random contains, in expectation, at least $(\frac{1}{2} + \epsilon)M$ edges, where M denotes the number of edges in any triangulation of P. From this we derive (to our knowledge) the first non-trivial upper bound of the form $c^n \cdot \mathrm{tr}(P)$ on the number of crossing-free geometric graphs on P; that is, at most a fixed exponential in n times the number of triangulations of P. This is joint work with Adreas Razen and Emo Welzl of ETH Zurich.