New Directions in Algorithms, Combinatorics, and Optimization

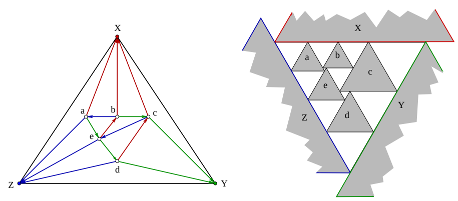
A Conference Honoring the 65th Birthday of William T. Trotter
May 5-9, 2008
Problem Session

---

1. (Jack Snoeyink) Given n disjoint line segments and m points in the plane, does there exist a piecewise-linear arc passing through all the points and crossing the line segments at most $2n-2$ times? The arc is only allowed to change direction at the given points.

2. (Alexander Kelmans) Does there exist an integer n such that every cubic, bipartite, cyclically n-connected graph has a Hamiltonian cycle? [The proposer proved that $n \geq 5$.]

3. (Stefan Felsner and Jan Kratochvíl) Does every 4-connected planar triangulation have a representation as a contact graph of internally disjoint homothetic triangles?



4. (Josh Cooper) Is it true that for every graph H there exists $c > 0$ such that if we pick uniformly at random an induced-H-free graph G with n vertices, then with high probability there exists a subset X of vertices of G of size at least cn such that X is independent or a clique.

5. (Doug West) Consider the following poset. The elements are subsets of $[n] = \{1, \ldots, n\}$. A set A covers another set B if A can be obtained from B either by adding the element 1 or by adding 1 to some element of B. Does this poset have a symmetric chain decomposition? (A partition of the elements into chains, where each chain is symmetric around the middle rank and does not skip any levels.)

6. (Joel Spencer) Start with the complete graph K_n, pick a random triangle and remove its edges (all three at once). Keep doing this until there are no triangles. How many edges are left? (Guess: $O(n^{3/2})$)

7. (Benny Sudakov) Take a graph with minimum degree strictly more than $\frac{3}{4}|V(G)|$. Then every maximum triangle-free subgraph is bipartite. [If this is true, then it is tight.]

8. (Stefan Felsner and Tom Trotter) Does the n-dimensional cube have a Hamiltonian path of the following form? Start with $\emptyset$. Whenever you visit a particular set, you must have visited all of its subsets with at most one exception.

9. (M. Reza Emamy) In the 5-cube, prove that at least 5 hyperplanes are needed to hit all edges, assuming you do not pass through any vertices. [In general, for the n-dimensional cube, the minimum number of hyperplanes is at least $\Omega(\sqrt{n})$ and at most $O(n)$.]

10. (Alexander Kelmans) Let $\lambda(G)$ be the maximum number of disjoint 3-vertex paths in G. Is it true that if G is cubic and 3-connected, then $\lambda(G) = \lfloor |V(G)|/3 \rfloor$? (The proposer proved that an affirmative answer to the above question would imply that Reed's conjecture (that the domination number of a cubic graph G is at most $\lceil |V(G)|/3 \rceil$) holds for cubic 3-connected graphs. Reed's conjecture is not true for cubic graphs of connectivity one [A. Kostochka and B. Stodolsky, DM 304 (2005) 749-762] or for cubic graphs of connectivity two [A. Kelmans, ArXiv: math.CO/0607512v1]).

11. (Harrison Brown) Let $d \geq 2$ be an integer, and let G be the graph with vertex-set $\mathbf{Z}^d$ with two vertices adjacent if they differ in every coordinate by at most one. Suppose we d-color the vertices of G, not necessarily properly. Does there exist a color such that the subgraph induced by that color has arbitrarily large connected components? [S. Norine and P. Seymour found a proof.]

12. (Doug West) Let $G = G_1 \cup G_2$, where G_1, G_2 have the same vertex-set, G_1 is a 2-regular graph and G_2 is the union of disjoint triangles. If the girth of G_1 is at least $\sqrt{|V(G)|}$, does there exist an independent set of size at least $|V(G)|/3$?

13. (Robin Thomas) Suppose that a graph G has $t - 5$ vertices whose deletion makes the graph planar. Then G has no K_t-minor. The converse is hopelessly false. But if G is t-connected and large as a function of t then the converse might be true.

14. (Paul Seymour) Let G be a graph with chromatic number at least t. Then Hadwiger's conjecture implies that there are t disjoint connected subgraphs pairwise touching. (Two subgraphs touch if either they share a vertex or there is an edge between them.) What about weakening Hadwiger's conjecture as follows. Is there a list of kt connected subgraphs pairwise touching such that every vertex is in at most k of them?

15. (Paul Seymour) Kostochka and Prince recently proved that for large t every graph with no $K_{3,t-2}$-minor is t-colorable. What about small t?