Triangle-Free Process

$$G_0, G_1, G_2, \ldots, G_M$$

M is a random variable

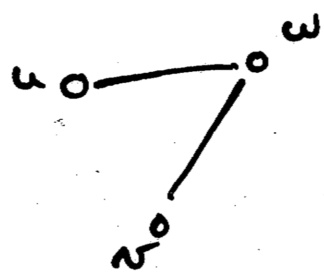
empty

random edge

$$G_{i+1} = G_i + e_{i+1}$$

$$E_i = E(G_i)$$

$$C_i = \{u,v\} \in \binom{[n]}{2} \quad \text{s.t. } \exists\, w$$

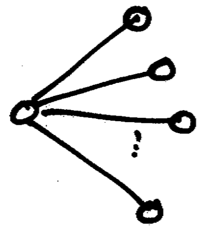


$\{u,v\}$ is <u>closed</u>.

$$O_i = \binom{[n]}{2} \setminus E_i \setminus C_i$$

e_{i+1} is chosen uniformly at random from O_i.

- Ruciński, Wormald

  forbid  $\}\ d$

- Erdös, Suen, Winkler $\triangle$-free process

  w.h.p. $\quad c_1 n^{3/2} \le M \le c_2 \log n \cdot n^{3/2}$

- Spencer $\triangle$-free process

  <u>conj.</u> $\quad M = \Theta\left(n^{3/2}\sqrt{\log n}\right)$ w.h.p.

- Kim $\quad \boxed{R(3,t) = \Omega\left(t^2/\log t\right)}$

- Bollobás, Riordan, Osthus, Taraz $\quad$ H-free process general H.

"beauty walks a razor's edge"

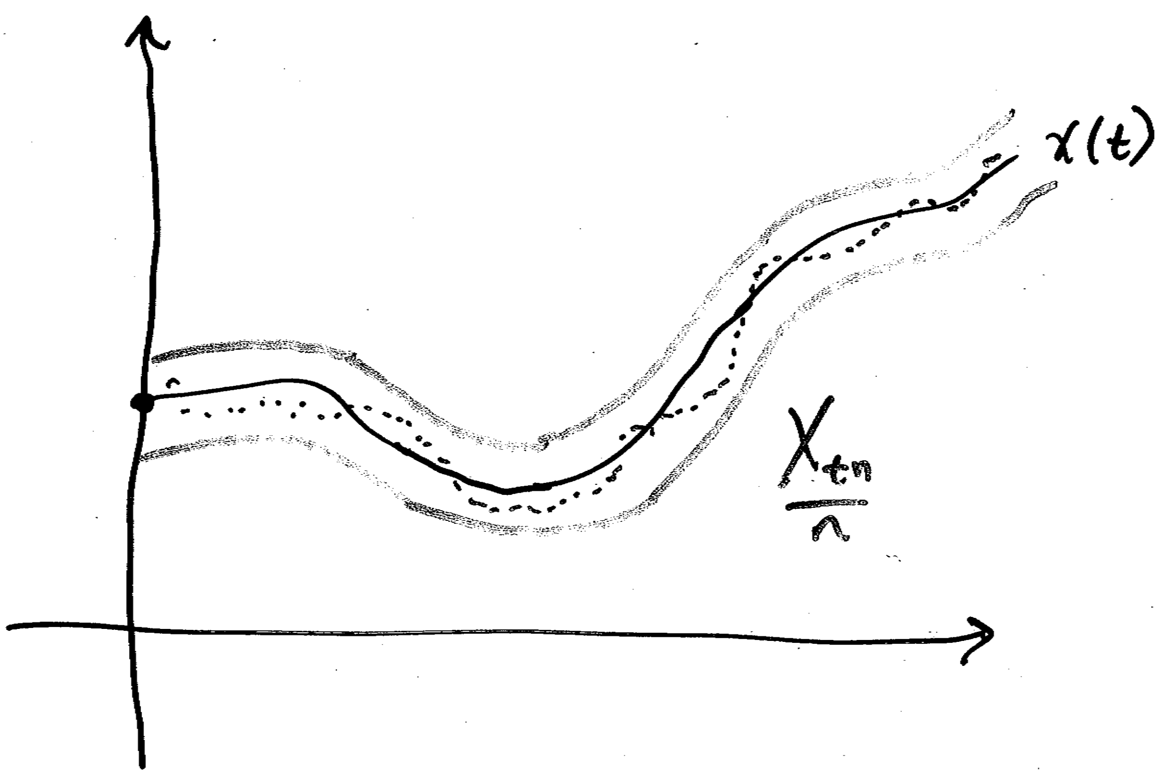
-Bob Dylan

$X_0, X_1, \ldots$ random variables

if (i) $E[X_{i+1} - X_i \mid \mathcal{F}_i] = f\left(\frac{X_i}{n}\right)$ and

(ii) $\frac{dx}{dt} = f(x) \qquad x(0) = \frac{X_0}{n}$

then $X_i \approx n\, x(i/n) \;\; \forall i$ w.h.p.



- The giant component

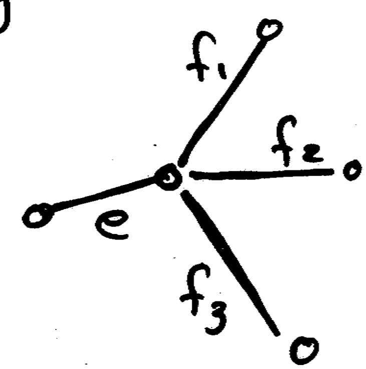
$e_1, f_1, e_2, f_2, e_3, f_3, \ldots$ random edges

step i : choose e_i or f_i.

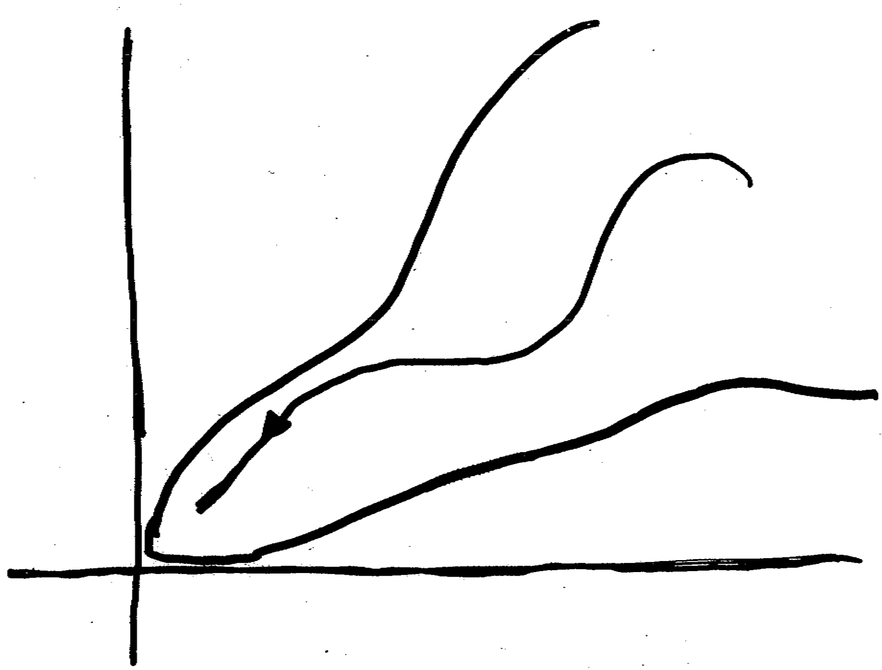
BK, Spencer Wormald, BBFP

- Karp-Sipser matching algorithm

take random edges with preference for pendant edges.



take e
delete
f_1, f_2, f_3

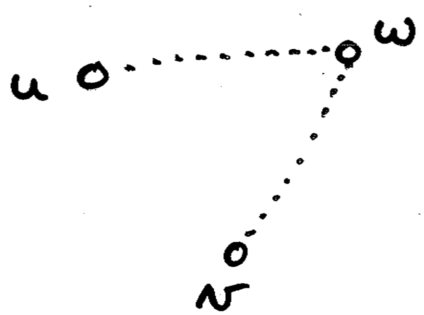


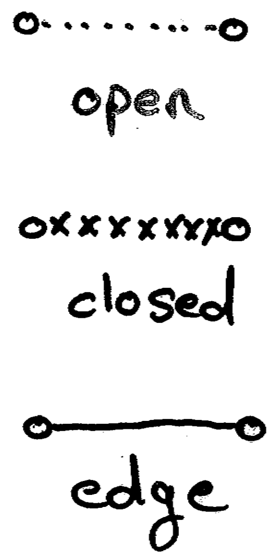
BF

random variables

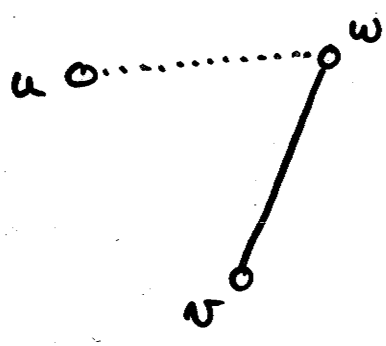
- $Q(i) = |O_i|$ = # of open pairs

- $X_{u,v}(i)$ = # w s.t.

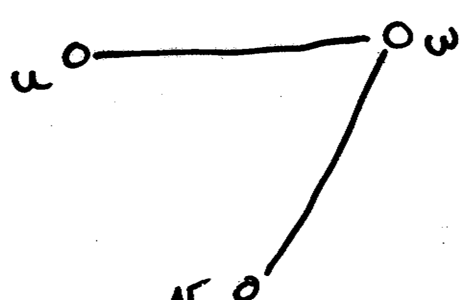




- $Y_{u,v}(i)$ = # w s.t.



- $Z_{u,v}(i)$ = # w s.t. = co-degree



Results

K_3-free process:

- $M = \Theta(n^{3/2}\sqrt{\log n})$
- $\alpha(G_M) = O(\sqrt{n \log n})$

$$\Rightarrow R(3,t) = \Omega(t^2/\log t)$$

Swallowing the process whole
sometimes provides better results
— Nick Wormald

K_4-free process:

- $M = \Theta(n^{8/5} \log^{1/5} n)$ with Peter Keevash
- $\alpha(G_M) = O(n^{2/5} \log^{4/5} n)$

$$\Rightarrow R(4,t) = \Omega(t^{5/2}/\log^2 t)$$