PERFECT MATCHINGS IN PLANAR CUBIC GRAPHS

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JOINT WITH

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LOVASZ – PLUMMER CONJECTURE, ~1975

$\exists\ c > 0$ SUCH THAT

EVERY 2-EDGE-CONNECTED CUBIC GRAPH

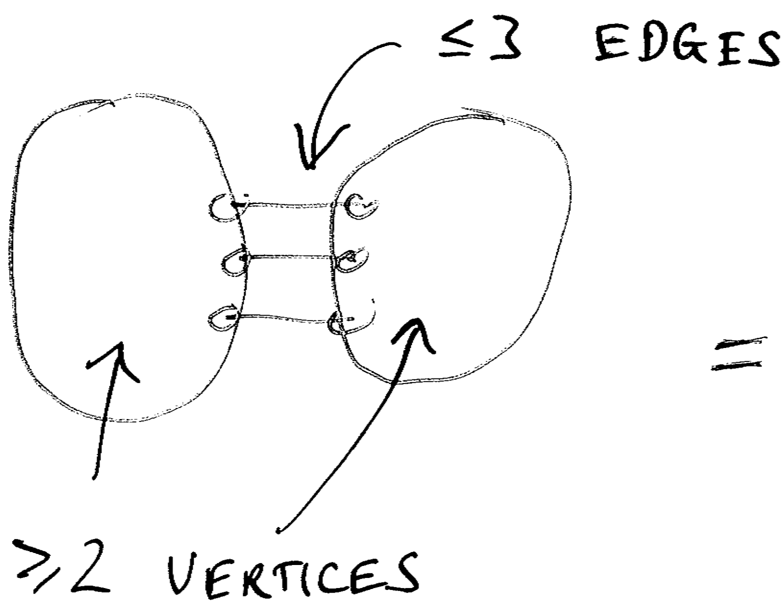
G HAS $\geq 2^{c|V(G)|}$

PERFECT MATCHINGS.

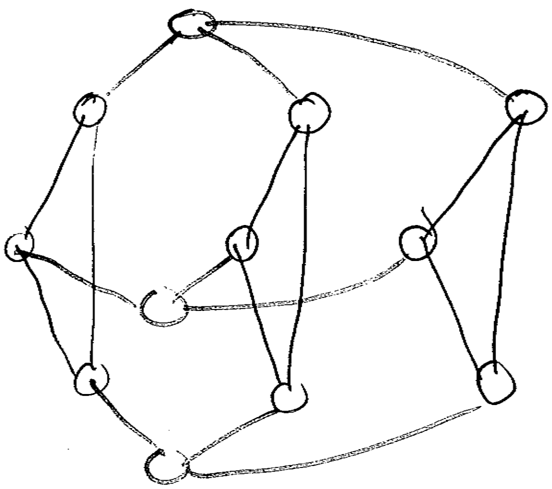
<u>THM</u> (LOVASZ – PLUMMER)

EVERY CYCLICALLY 4-CONNECTED CUBIC GRAPH G HAS $\geq \frac{1}{2}|V(G)| + 1$

PERFECT MATCHINGS.



= NOT CHC



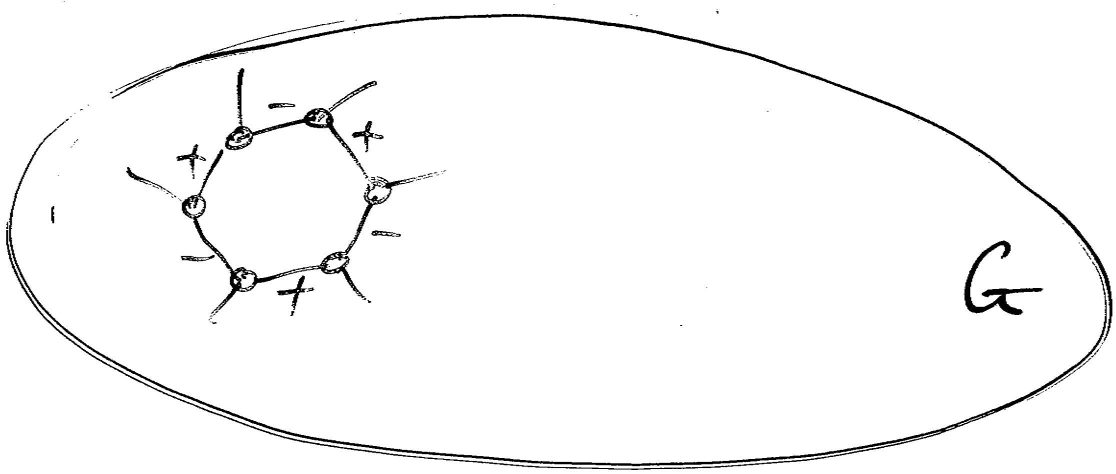
ONLY 6 PERFECT MATCHINGS

KRAL 2008	EVERY 2-EDGE-CONNECTED CUBIC GRAPH G HAS $\geq \frac{1}{2}\lvert V(G)\rvert$ PERFECT MATCHINGS

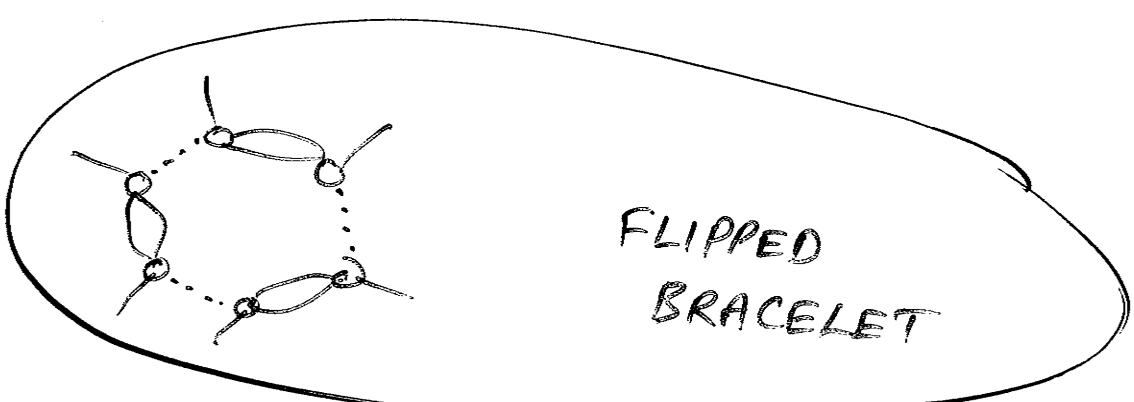
THM EVERY 2-EDGE-CONNECTED PLANAR CUBIC GRAPH G HAS

$$\geq 2^{|V(G)|/655978752}$$

PERFECT MATCHINGS.



BRACELET



LOOK FOR: SET OF DISJOINT BRACELETS SUCH THAT FLIPPING ANY SUBSET OF THEM GIVES GRAPH WITH NO CUTEDGE

BLING

$\beta(G)$ = SIZE OF LARGEST SUCH SET

IN PLANAR G,

$\#$ PERFECT MATCHINGS $\geq 2^{\beta(G)/3}$

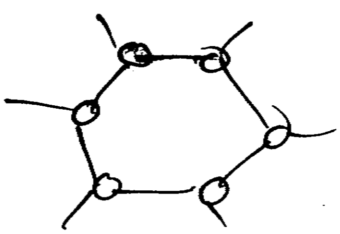
THM IF G IS PLANAR CUBIC, WITH NO CUTEDGE, THEN

$$\beta(G) \geq \frac{|V(G)|}{218659584}$$

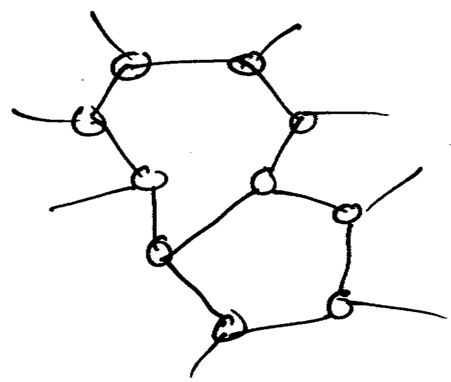
THM: IF G IS C4C PLANAR, THEN $\beta(G) \geq \dfrac{|V(G)|}{30976}$.

$|V(G)| = N$. # REGIONS $= \frac{N}{2} + 2$.

DOMINO: EVEN CYCLE BOUNDING ≤ 2 REGION.



OR

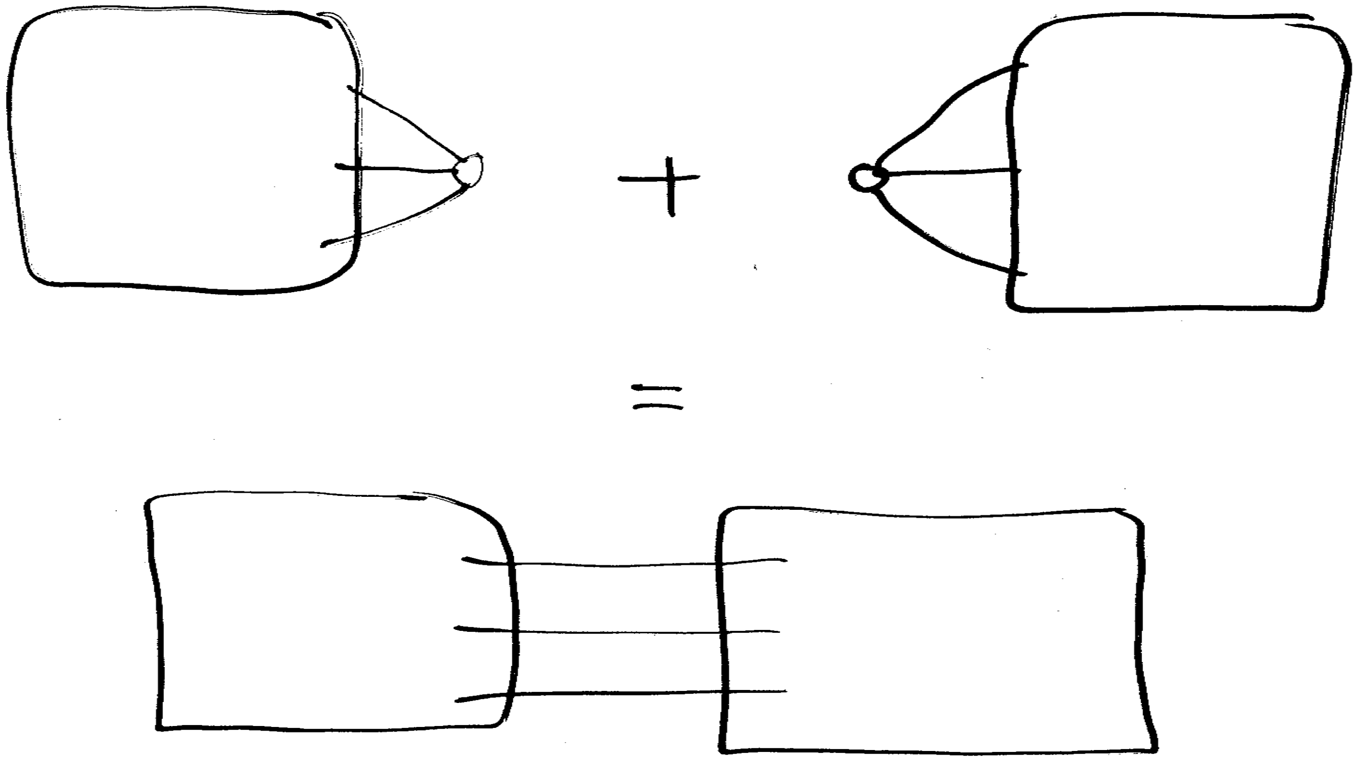


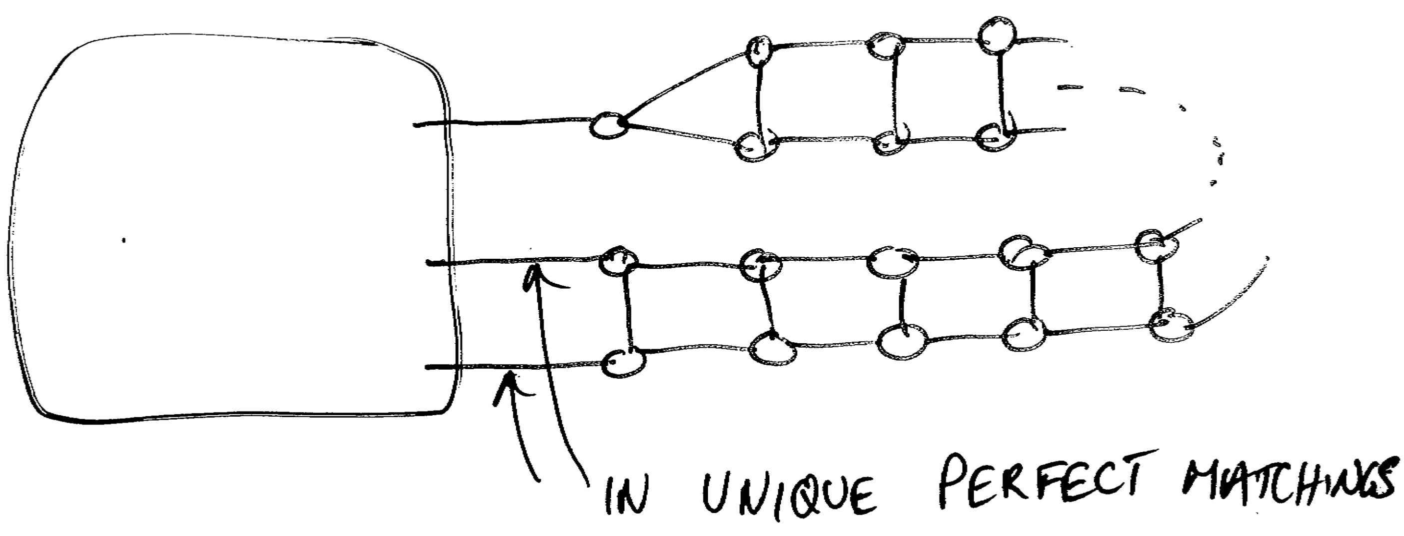
$\frac{N}{4}$ DOMINOES.

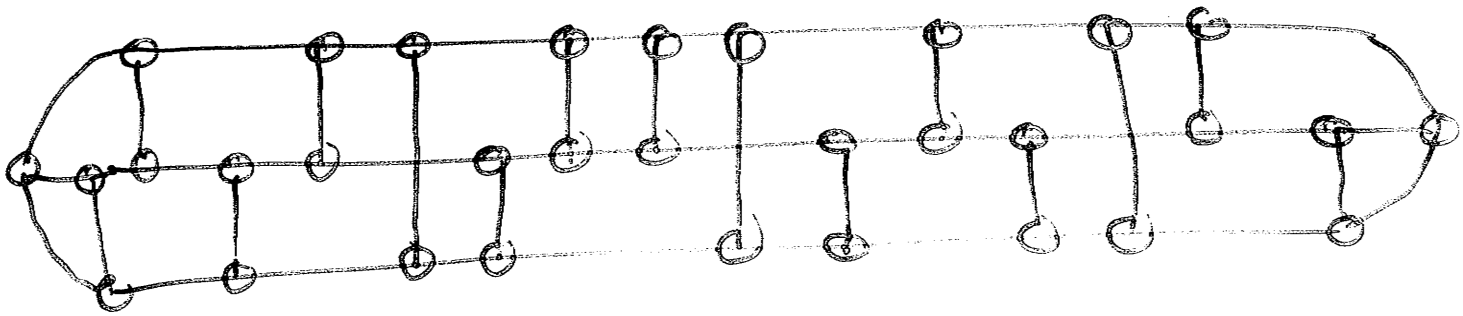
$\frac{N}{8}$ ALL OF LENGTH ≤ 15

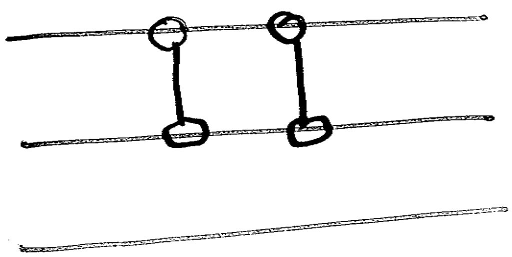
$\frac{N}{32}$ PW VERTEX-DISJOINT

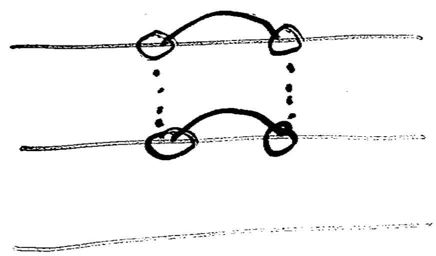
THM IF G IS 3-CONNECTED PLANAR, THEN $\beta(G) \geq \frac{|V(G)|}{16819968}$

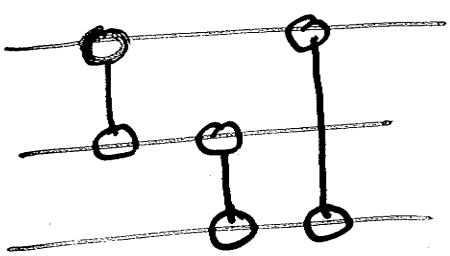


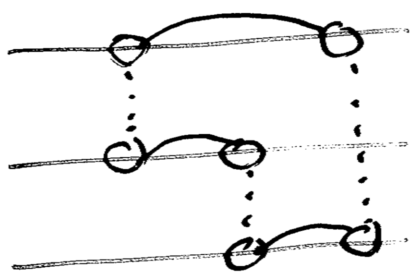


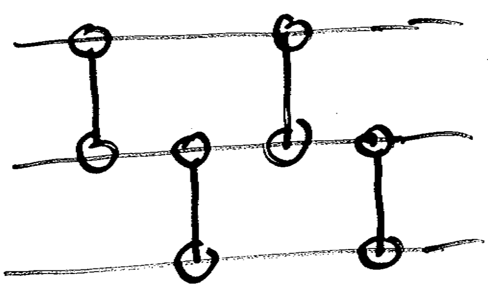


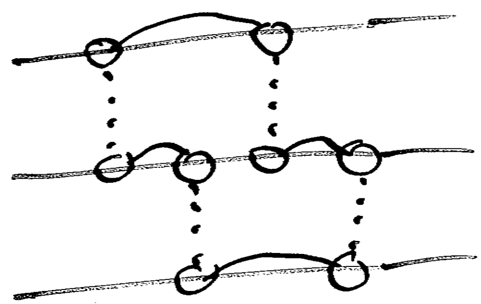






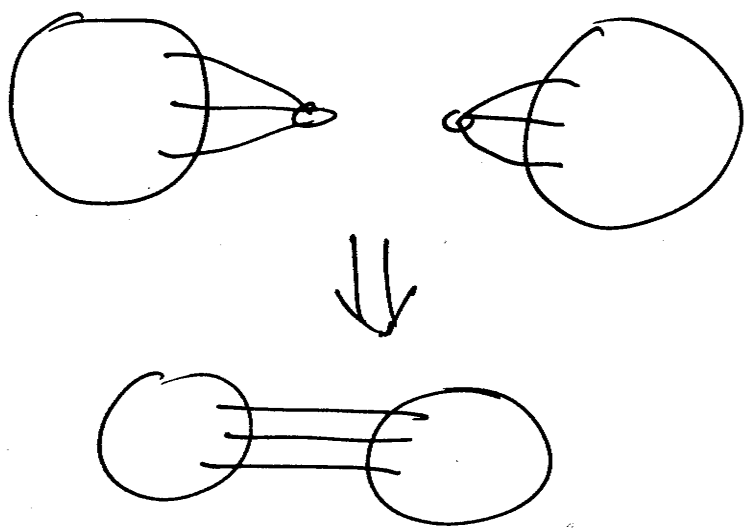


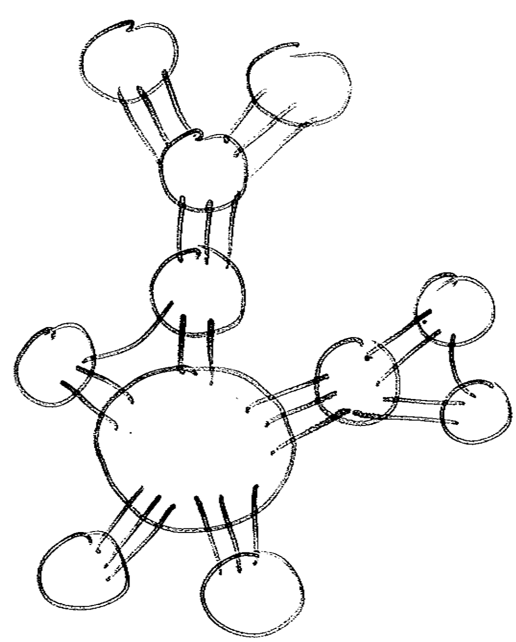




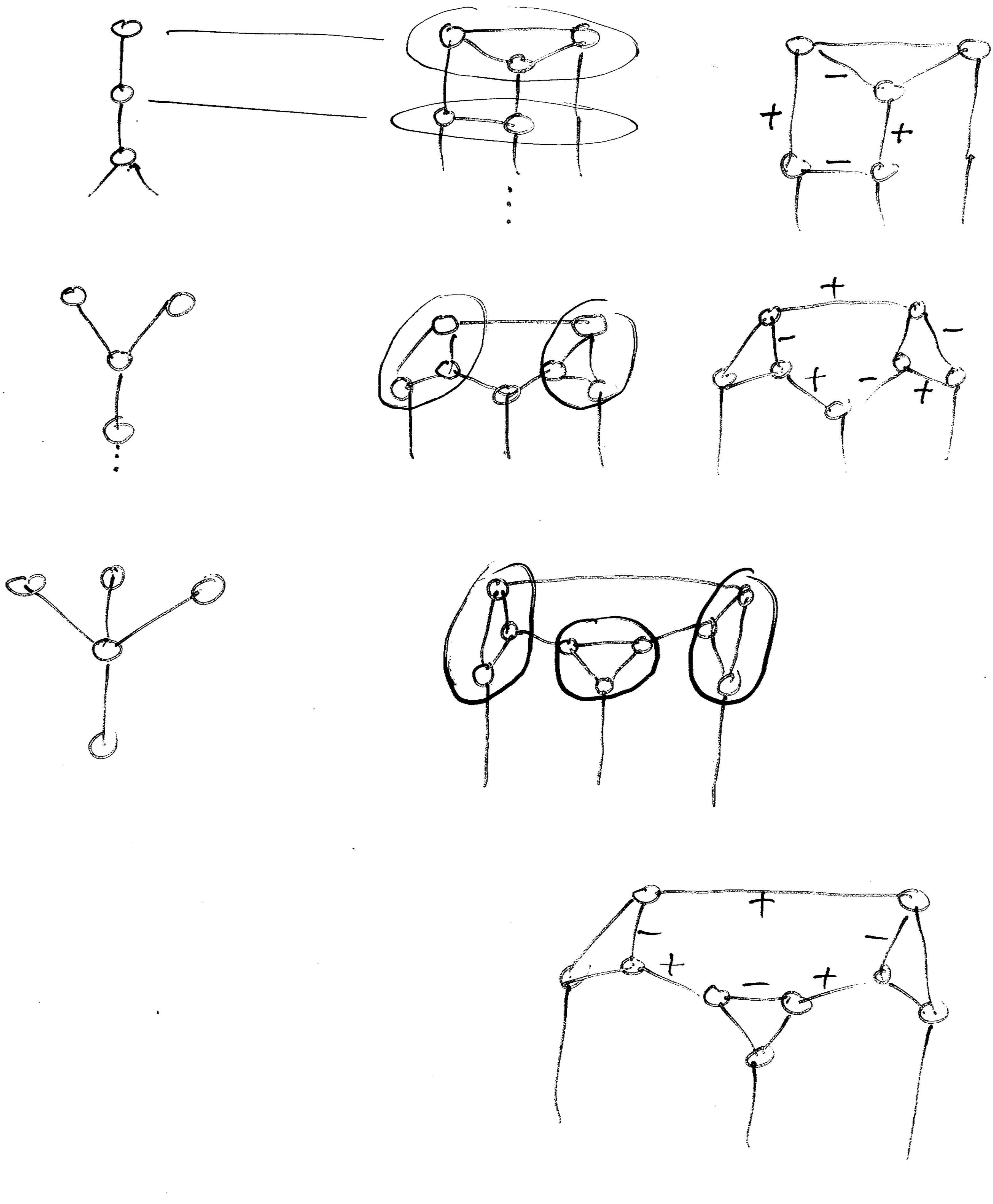
THM (FOWLER &, 1998)

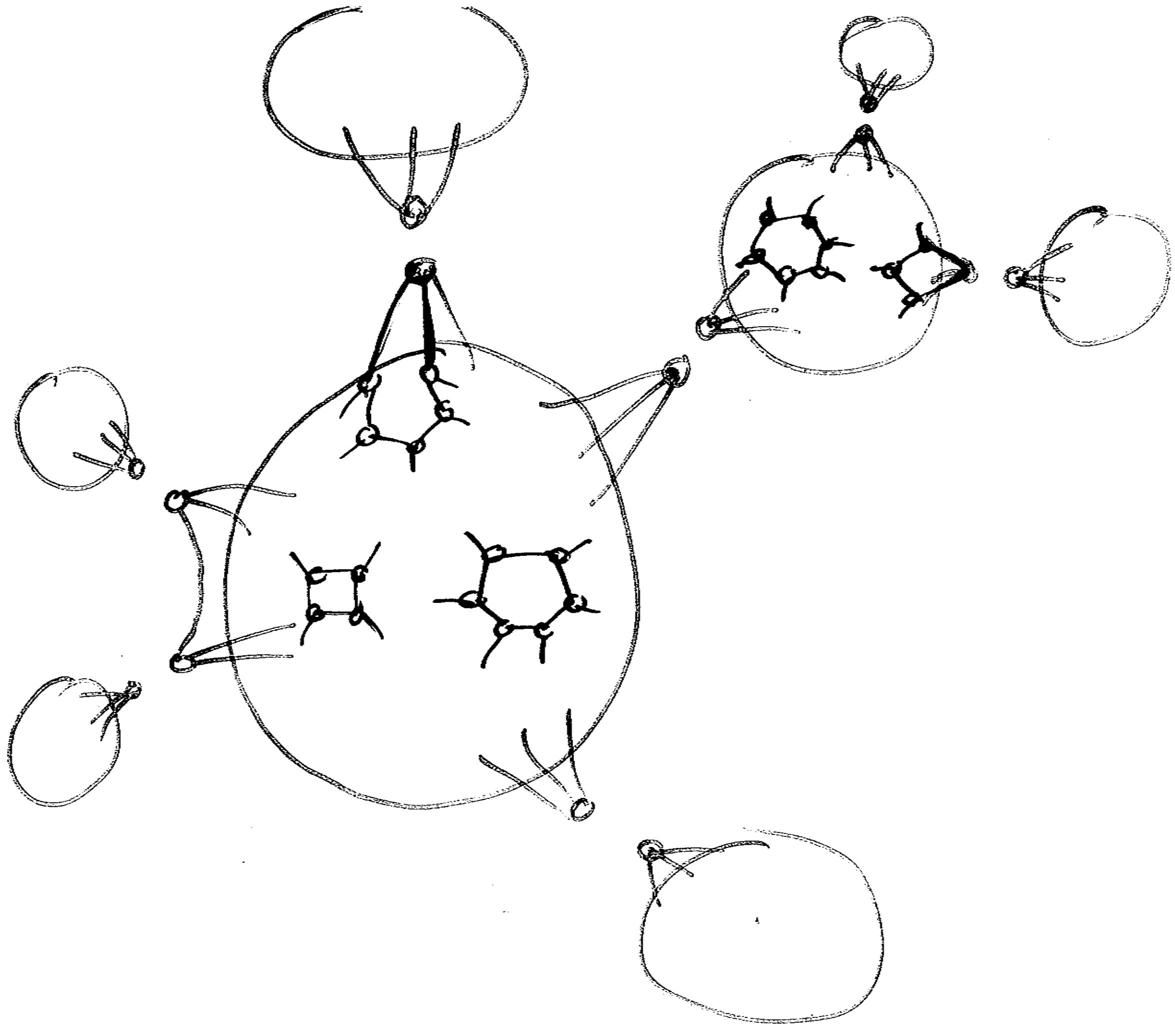
K_4 IS THE ONLY C4C PLANAR CUBIC GRAPH THAT IS UNIQUELY 3-EDGE-COLOURABLE.





LEAVES OF FOWLER TREE :





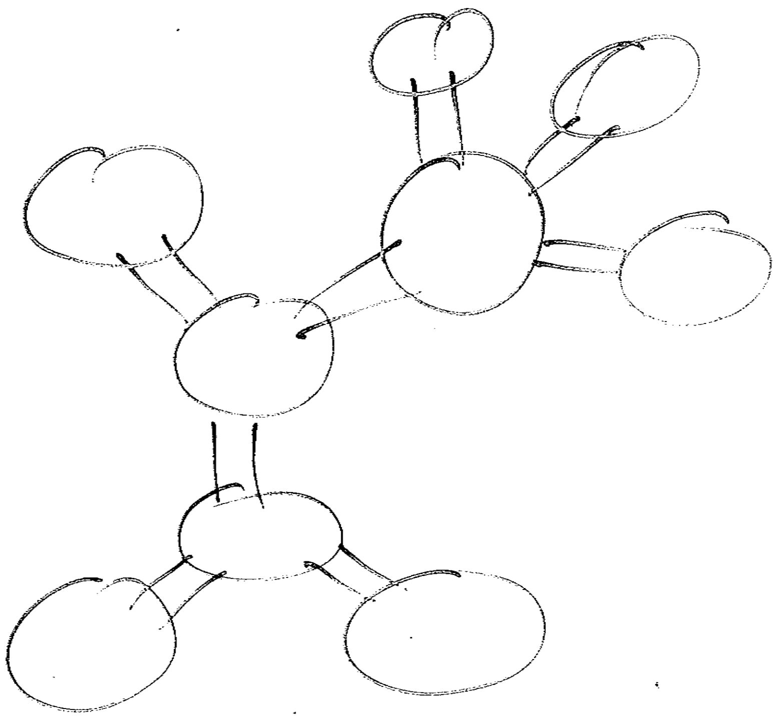
CASE 1 : NO OF CUTS $= o(|V(G)|)$

CASE 2 : $O(|V(G)|)$ PIECES ARE NOT UNIQUELY 3-COL.

CASE 3 : ALMOST FOWLER ..... ??

$\boxed{\text{REORGANIZE}}$: FIRST, CONTRACT ALL TRIANGLES. THEN CASES 1, 2, 3

2 - CONNECTED CASE



CASE 1: NO OF CUTS $o(|V(G)|)$

CASE 2: $O(|V(G)|)$ CUTS ALL WITH SAME COLOUR.